

### **AMENDMENTS TO THE CLAIMS**

This listing of claims will replace all prior versions, and listings, of claims in the application:

#### **Listing of Claims:**

1-27. (Canceled)

28. (New) A method for multi-objective portfolio optimization, the method comprising the steps of:

generating an initial population of solutions of portfolio allocations using a combination of linear programming and sequential linear programming algorithms in a portfolio configuration space using a computing device, the portfolio configuration space having a plurality of dimensions;

generating a first interim efficient frontier in a portfolio performance space having at least three dimensions using a Pareto Sorting Evolutionary Algorithm (PSEA);

generating a second interim efficient frontier in the portfolio performance space using a Target Objectives Genetic Algorithm (TOGA);

concatenating the first interim efficient frontier with the second interim efficient frontier to create a third interim efficient frontier; and

passing the third interim efficient frontier through a dominance filter to generate a final efficient frontier for use in investment decisions.

29. (New ) The method of Claim 28, further comprising the steps of:

generating a non-dominated solution set comprising an efficient frontier in the portfolio performance space using one of an evolutionary algorithm and optimization processing;

imposing a sequence of user-specified constraints in at least one of the portfolio performance space and the portfolio configuration space to reduce solutions in the non-dominated solution set to an initial solution subset; and

executing a sequence of Pareto filters in a user-specified order on regions of a lower dimensional portfolio performance space having fewer dimensions than the portfolio performance space to produce a resulting solution subset having a fewer number of points than the initial solution subset, the resulting solution subset being used in investment decisions.

30. (New) The method of claim 29, wherein the user-specified constraints is defined by limits on performance metrics.

31. (New) The method of claim 30, wherein the performance metrics include risk and return.

32. (New) The method of claim 29, wherein the user-specified constraints include imposing a lower limit on return and an upper limit on risk.

33. (New) The method of claim 29, wherein the user-specified constraints include imposing a first range on return and a second range on risk.

34. (New) The method of claim 29, further comprising the step of:  
applying additional user-specified constraints to the resulting solution subset to produce a final selection.

35. (New) The method of claim 29, further comprising the step of:  
imposing preferences on the resulting solution subset to produce a final selection.

36. (New) The method of claim 35, wherein the preferences are represented by relative weights on performance metrics.

37. (New) The method of claim 35, wherein the preferences are represented by relative weights on performance configuration metrics.

38. (New) The method of claim 29, wherein after the imposing step, the method further includes:

applying portfolio configuration metrics based on a plurality of asset allocations in a portfolio; and

comparing portfolio configuration metrics between a plurality of portfolios.

39. (New) The method of claim 38, wherein the comparing step includes determining a required transaction to transform the plurality of asset allocations in a currently existing portfolio to a plurality of asset allocations in each of the portfolios in the resulting solution subset.

40. (New) The method of claim 29, wherein the user-specified constraints are one of independent and dependent constraints.

41. (New) The method of Claim 28, further comprising the steps of:  
identifying at least one region having a gap in the at least three-  
dimensions of the final efficient frontier using a visualization tool;  
interactively placing at least one target in the at least one region using  
the visualization tool; and  
generating supplemental solutions to the final efficient frontier using the  
Target Objectives Genetic Algorithm (TOGA) to create a second final efficient frontier,  
the second final efficient frontier being used in investment decisions.

42. (New) The method of claim 41, further including the step of selecting at least one portfolio from the second final efficient frontier.

43. (New) The method of claim 41, wherein the TOGA further includes the steps of:

accepting a set of target vectors; and  
generating a series of chromosomes, evaluated on the basis of the  
accepted target vectors, over multiple generations.

44. (New) The method of claim 43, wherein the TOGA further includes the step of evaluating a fitness of each chromosome until a population with an acceptable fitness is determined so as to fill in the gap.

45. (New) The method of claim 41, wherein the gap is a region that is sparsely populated by possible solutions.

46. (New) The method of Claim 28, wherein generating the first interim efficient frontier in the portfolio performance space comprises the steps of:

- (a) randomly drawing from the initial population of solutions of portfolio allocations;
- (b) passing the initial population of solutions of portfolio allocations through the dominance filter to identify a non-dominated subset of parent portfolio allocations;
- (c) committing the non-dominated subset of parent portfolio allocations to a non-dominated portfolio allocations archive;
- (d) randomly combining matched pairs of parent portfolio allocations to create offspring portfolio allocations;
- (e) passing the offspring portfolio allocations through the dominance filter to identify a non-dominated subset of offspring portfolio allocations;
- (f) combining the non-dominated subset of parent portfolio allocations with the non-dominated subset of offspring portfolio allocations into a larger set of portfolio allocations;
- (g) passing the larger set of portfolio allocations through a non-crowding filter to identify a reduced subset of portfolio allocations;
- (h) creating a new population of individual portfolio allocations from the reduced subset of portfolio allocations;
- (i) updating the non-dominated portfolio allocations archive with the new population of individual portfolio allocations;
- (j) repeating steps (a) through (i) for a plurality of generations; and
- (k) passing the updated non-dominated portfolio allocations archive through the dominance filter to generate the first interim efficient frontier in the portfolio performance space.

47. (New) The method of Claim 46, wherein the non-dominated subset of parent portfolio allocations has a first cardinality.

48. (New) The method of Claim 47, wherein the non-dominated subset of offspring portfolio allocations has a second cardinality that is different than the first cardinality.

49. (New) The method of Claim 48, wherein the larger set of portfolio allocations has a third cardinality that is equal to the first cardinality plus the second cardinality.

50. (New) The method of Claim 49, wherein the reduced subset of portfolio allocations has a fourth cardinality that is less than the third cardinality.

51. (New) The method of Claim 46, wherein steps (a) – (j) are repeated until convergence is achieved or allocated computational cycles are exhausted.

52. (New) The method of Claim 46, wherein the reduced subset of portfolio allocations has a fourth cardinality, and wherein the new population of individual portfolio allocations is exactly equal to the reduced set of portfolio allocations if the fourth cardinality is equal to the initial population of individual portfolio allocations.

53. (New) The method of Claim 46, wherein the reduced subset of portfolio allocations has a fourth cardinality, and wherein the new population of individual portfolio allocations is created by randomly drawing additional individual portfolio allocations from the portfolio allocations archive if the fourth cardinality is less than to the initial population of individual portfolio allocations.

54. (New) The method of Claim 46, wherein the reduced subset of portfolio allocations has a fourth cardinality, and wherein the new population of individual portfolio allocations is created by randomly discarding individual portfolio allocations from the reduced subset of portfolio allocations if the fourth cardinality is greater than to the initial population of individual portfolio allocations.

55. (New) The method of Claim 54, wherein the new population of individual portfolio allocations is created by randomly injecting individual portfolio allocations from the portfolio allocations archive until the fourth cardinality is equal to a desired number of individual portfolio allocations.

56. (New) The method of claim 28, wherein the step of passing the third interim efficient frontier through the dominance filter comprises the steps of:

(a) selecting a first dimension from the at least three dimensions of the portfolio performance space;

- (b) generating bins for all remaining non-selected dimensions of the portfolio performance space;
- (c) determining a solution in each bin of the non-selected dimensions with a maximum value along the selected dimension;
- (d) comparing the solution with the maximum value in each bin to other solutions in each bin to determine whether the other solutions are dominant solutions or dominated solutions; and
- (e) removing the dominated solutions from the portfolio performance space so as to result in a reduced set of solutions, the reduced set of solutions being used in investment decisions.

57. (New) The method of claim 56, the method further including the step of repeating steps (a) – (e) for at least a second dimension of the portfolio performance space after the dominated solutions are removed from the portfolio performance space.

58. (New) The method of claim 56, wherein the plurality of dimensions is  $n$ -dimensions, and the bins are in the form of  $n-1$  dimensional polyhedra in the portfolio performance space.

59. (New) The method of claim 56, further including the step of performing a final dominance check on the reduced set of solutions.

60. (New) The method of claim 59, wherein the step of performing the final dominance check on the reduced set of solutions includes generating an efficient frontier.

61. (New) The method of claim 56, wherein the investment decisions are based on competing objectives that include risk and return.

62. (New) The method of claim 56, further including the step of repeating steps (a) – (e) for all remaining dimensions of the portfolio performance space after the dominated points are removed from the portfolio performance space.

63. (New) The method of claim 62, wherein a coarseness of the bins is decreased as remaining dimensions of the portfolio performance space are selected.

64. (New) The method of claim 56, wherein the step of comparing the solution with the maximum value in each bin to other solutions in each bin includes using Pareto dominance that includes uncertainties in measuring competing objectives.

65. (New) A system for multi-objective portfolio optimization, the system comprising:

- a population generation portion that generates an initial population of solutions of portfolio allocations using a combination of linear programming and sequential linear programming algorithms in a portfolio configuration space, the portfolio configuration space having a plurality of dimensions;

- a first processing portion that generates a first interim efficient frontier in a portfolio performance space having at least three dimensions using a Pareto Sorting Evolutionary Algorithm (PSEA);

- a second processing portion that generates a second interim efficient frontier in the portfolio performance space using a Target Objectives Genetic Algorithm (TOGA);

- a fusion portion that concatenates the first interim efficient frontier with the second interim efficient frontier to create a third interim efficient frontier; and

- a dominance filtering portion that generates a final efficient frontier from the third interim efficient frontier for use in investment decisions.

66. (New) A computer readable medium for multi-objective portfolio optimization, the computer readable medium comprising:

- a first portion that generates an initial population of solutions of portfolio allocations using a combination of linear programming and sequential linear programming algorithms in a portfolio configuration space, the portfolio configuration space having a plurality of dimensions;

- a second portion that generates a first interim efficient frontier in a portfolio performance space having at least three dimensions using a Pareto Sorting Evolutionary Algorithm (PSEA);

- a third portion that generates a second interim efficient frontier in the portfolio performance space using a Target Objectives Genetic Algorithm (TOGA);

a fourth portion that concatenates the first interim efficient frontier with the second interim efficient frontier to create a third interim efficient frontier; and

a fifth portion that generates a final efficient frontier from the third interim efficient frontier for use in investment decisions.